THE TEACHING OF MATHEMATICS IN THE JUNIOR HIGH SCHOOL.*

(A Discussion of the Rochester Plan.)

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Introduction.

Any consideration of so extensive a topic presupposes a clear statement of the method by which one expects to disentangle the maze of interacting factors which seem to underlie the situation to be discussed. Undoubtedly there are several ways of approaching the question that is before us. The controlling idea in the mind of the writer, however, was the necessity of putting

* This paper is intended as a summary of several preliminary reports presented at meetings of the Rochester Section and at the Trenton meeting of the Association, April 28, 1917. It is based on a report submitted to the school authorities of Rochester. For the sake of brevity the discussion emphasizes only the academic course. A differentiated curriculum in mathematics is in operation for pupils in the commercial course and in the industrial and household arts courses.

into bold relief, if possible, those elements which above all others are significant in the evolution of a satisfactory mathematical curriculum.

In former years investigations of this type were conducted almost entirely from a pedagogic standpoint. If schoolroom results were poor, it was customary to attack the teacher, or the textbook, or—more rarely—the examination. The curriculum was stoically accepted as something essentially unalterable. Pupils were treated to a fixed menu of courses, of topics and methods, until their increasing mortality called imperatively for remedial measures. Very gradually it has dawned upon educational authority that democratic education, especially in our big American cities, presents such a multiplicity of embarrassing aspects that an offhand, machine-made solution, of which we are usually so fond, is entirely out of the question. Only genuine, careful experimentation can be of avail. It is a pleasure to note that Rochester has officially taken this sane attitude. After two years of experience in the new junior high school the attempt is to be made to offer a first tentative summary of experiences and results, and thus to obtain, if possible, a revised and improved program for the next forward movement.

As has been suggested, the question of mathematics in the junior high school is by no means exclusively a pedagogic one. Innumerable factors over which the teacher has no control, factors which are essentially administrative or social, often outweigh in importance all other considerations. Hence the report which is to follow will have to give due attention to at least three phases of the problem.

I. The Pedagogic Elements of the Question.

1. *Aim.*—It is doubtful whether any other educational item has received more careful attention in the last fifteen years than the teaching of mathematics. In 1912 the United States Bureau of Education published a special bulletin containing a bibliography of 1,849 titles, all pertaining to books, memoirs, and pamphlets

* The Washington Junior High School opened its doors in September, 1915.

written since 1900, either here or abroad, on the teaching of mathematics. An international commission was created to unify and standardize the mathematical teaching of all the civilized countries. As a result, scores of valuable treatises and reports have come to us. Surely, a subject of such vitality, at once the oldest, the most progressive, and the most far-reaching, of all the sciences, deserves serious consideration, on purely a priori grounds, in the curriculum of any school.

Nevertheless, mathematics cannot be said to be an extremely popular subject. This is due to several reasons: (1) rigorous thinking, at best, is not a pleasant task to most people; (2) the results of mathematics are of a cumulative character, *i. e.*, the subject must be studied in a strictly progressive order, and therefore losses due to absence or neglect often cause failure; (3) the teaching of the subject has failed to emphasize the inexhaustible range of mathematical applications, but has given almost exclusive attention to theory; (4) the brief period of time allowed for teaching the subject has made it impossible to cover more than the fundamentals, and even these in a very perfunctory way, thus giving a wholly distorted and inadequate conception of the purpose and scope of mathematics.

Hence the anomaly has arisen, under the influence of a socialized pedagogy calling for immediate educational returns, that mathematics, the oldest and most indispensable of all sciences, has been driven into a defensive position. The mother of all sciences is requested to *justify* her appearance in the class room. Why ask every boy and girl to give "so much valuable time" to this terrible, uninteresting, dry-as-dust array of symbols and formulas? The mathematical apologist, therefore, has before himself the inviting task of explaining "what there is in mathematics that the working man needs and what of this science the woman requires in directing the education of her children and in managing her home; how mathematics trains the mind and how its poetry affects human life; what potency the subject has in the uplift of the soul of you and of me; how it has linked itself with all humanity in all times; and how we should go about to make all this real in our everyday teaching."

Fortunately, able pens have taken up this defense. An increasing volume of literature is becoming available to all who

take an interest in this problem. Even the layman will derive both benefit and pleasure by reading such excellent summaries as those of Professor C. J. Keyser,* of Columbia University, and of Professor D. E. Smith,† of Teachers College. He will find that to well-informed persons this agitation against mathematics is amusing, because it is not only useless, but also without point. He will find, moreover, that honest critics distinguish sharply between mathematics as a science and the pedagogy of the subject. It is granted without a moment of hesitation that much, very much, remains to be done to make mathematical teaching what it may and should be. There is ample evidence, nevertheless, that the criticisms constantly directed against the teaching of mathematics may with equal justice be brought against any other school subject. Does history function, or literature, or music, or penmanship, or biology? If not, why teach them? The absurdity of such a position forces a candid investigator to confess that our educational shortcomings are not necessarily inherent in the subject-matter taught, but rather in the whole intricate educational problem of the twentieth century.

It cannot be denied, however, that the insistent attacks on mathematics during recent years have had the extremely valuable effect of focusing attention on the aim of all mathematical instruction. For if no satisfactory reason can be given for including this subject in the curriculum of a junior high school, it should certainly be dropped.

Briefly stated, there are three main reasons for asking boys and girls to pursue at least an elementary course in mathematics.

First, mathematics is the basis of practically our entire material civilization. Without it we should return to primitive conditions. To an extent which it is almost impossible to exaggerate, our entire business life, including manufacturing and transportation, our engineering enterprises, architecture, sciences such as physics, chemistry, economics, astronomy, statistics, and

* C. J. Keyser, "The Human Worth of Rigorous Thinking," Columbia University Press, 1916.

† D. E. Smith, "Mathematics in the Training for Citizenship," *Teachers College Record,* May, 1917.

even psychology, are absolutely dependent on the ideas and processes of mathematics.

The marvelous shorthand of algebra, which "says more in fewer words than any other language," and the mensurational power of geometry, enter in so many ways into the world's ordinary affairs that a boy or girl ignorant of the first principles of mathematics would be handicapped both from a standpoint of general culture and of life preparation. Hence it is undeniable that on the basis of practical utility alone the teaching of mathematics can be fully justified.

A more difficult task is encountered if, secondly, we attempt to explain the usual mathematical curriculum by means of the customary argument that it offers an unparalleled opportunity for mental training. No other contention has been debated more fervently during recent years. It seems that the whole matter is largely a matter of phraseology. As the smoke of battle is lifting, it is becoming clearer and clearer that the famous doctrine of mental discipline has not been disproved and never will be disproved, and that both its extreme advocates and its extreme opponents are wrong. Whether one prefers Thorndike's* conception of "identical elements," or Judd's convincing theory of generalized experience, the truth is that "no one denies the fact of transfer of training." The following quotations from Judd† have a particular bearing on our subject:

"The real questions at issue are what is the degree of transfer and what is its method" (p. 405).

"The negative evidence offered by the critics of formal discipline thus turns out to be of the thinnest possible type. They have not only not proved a negative; they have presented a series of facts which calls loudly for affirmative discussion" (p. 412). "Formalism and lack of transfer turn out to be not characteristics of subjects of instruction, but rather products of the mode of instruction in these subjects" (p. 413). "The

* Thorndike, E. L., "Educational Psychology," 1913, Vol. II., Chap. XII., especially pp. 430 and 431.

See also Ruediger, W. C., "The Principles of Education," Houghton Mifflin Co., 1910, Chap. VI.

† Judd, Ch. H., "Psychology of High School Subjects," Ginn & Co., 1915, pp. 392–435.

problem of education thus turns out to be the problem of generalizing experience" (p. 419). "The generalizing of experience is a qualitatively new fact wherever it appears" (p. 420). "A subject which gets itself so organized that it rotates around its own center immediately becomes formal. . . . The opposite of formalism is not emphasis on content, but emphasis on application. Any content may become formal. Any mode of procedure may become formal. Opposed to 'formal' is such a word as 'vital.' That is not formal which moves forward to new applications. Generalized knowledge is not formal. Knowledge which is being used in applications, either in the evolution of higher thought-processes or in the solution of practical problems, is not formal" (p. 421). "Application is, however, a most difficult mental process, and needs to be learned just as the original principle itself has to be learned. . . . Successive examples, therefore, should be treated as the opportunity on the part of the teacher to cultivate the attitude of application" (p. 422). "The teacher ought to recognize in all of these cases that the mere solution of the problem is of slight importance if the student does not acquire that higher power of discovering the mode of procedure which is appropriate in each case" (p. 423). "Those who have opposed the doctrine of formal discipline by saying that the school subjects at the present time do not give a generalized training are undoubtedly criticizing not the human mind, but our methods of instruction" (p. 424). "There is no adequate justification for the loud contention of the newer subjects that the older subjects of the curriculum are inherently formal and of necessity narrow in the effects which they produce on students' minds" (p. 425). "The mastery of the general principle is therefore a new type of mental achievement. . . . The discovery of this general principle or rule is a new performance; it is an expression of the power of generalization. The cultivation of this power of generalization is the most important achievement in the student's education. It will not come without special endeavor on the part of the student and on the part of the teacher" (p. 432).

Let any one attempt to find a subject in the whole curriculum to which Judd's ideas of generalized training could be more easily applied than in the field of mathematics. Since in our

everyday experiences we are constantly encountering quantitative relations and problems, and since our thinking is very largely deductive, it is evident that training in mathematics, if it is properly conducted, *must* carry over into the affairs of life. Surely no other subject in the curriculum offers more points of contact between theory and practice than does mathematics. It is no accident that for four thousand years this subject has demanded attention with increasing persistence. We may no longer be able to cultivate in the minds of boys and girls that sheer enthusiasm for geometry, rhetoric, music and the other fine arts, which characterized the education of Greece during the period of her greatest splendor. We may never again desire, like Pythagoras and Plato of old, to refuse admission to the gates of the educational palace to any one unacquainted with mathematical learning. It is, however, the combined testimony of innumerable witnesses that the study of mathematics, under adequate guidance, will and does leave upon the mind an indelible impress of permanent value, a testimony which is not shattered by the recent literature of mental discipline.*

From a utilitarian standpoint, therefore, mathematics ranks in importance with reading and writing, while its "disciplinary value" is not exceeded by that of any other subject. Perhaps it is unfortunate that this should be so. For this very fact has served to overshadow completely the third aim of mathematical studies, undoubtedly the most fundamental of all. Neither practical considerations nor formal discipline alone have caused the gigantic development of mathematics, either past or present. Behind it all was a far deeper impulse. Is it chance that the list of the world's noted mathematical thinkers includes the greatest philosophical geniuses of all time, men like Pythagoras, Plato, Descartes, Leibnitz? Or is it accident that the *mathematical* university of Cambridge, England, trained many of England's greatest poets, and not *literary* Oxford? It was

* Teachers of mathematics are greatly indebted to Dr. H. O. Rugg for his careful study, from a mathematical standpoint, of this vexing problem. It is a pleasure to refer to his recent monograph: "Experimental Determination of Mental Discipline in School Studies," Warwick and York, 1916. His conclusions are distinctly affirmative (see pp. 114–116). The book also contains a valuable chronological bibliography.

a poet who asserted that "the life of the gods is mathematics." Said Plato, "God always geometrizes;" and Gauss, that prince of modern mathematicians, exclaimed: "God always arithmetizes."

What is mathematics? It is difficult, if not impossible, to give a brief and concise definition of this many-sided branch of learning. The mathematical sciences have ever maintained a close relation to philosophy. They have assisted in the continual endeavor to explain this world of a myriad ideas and of countless bewildering phenomena as a unified system controlled by law. The mathematician really tries to reduce all thinking to its ultimate bases and to comprehend the physical universe by means of a few fundamental concepts such as mass, energy, space and time. In short, it is the passion for infinite harmony in a world of apparent chaos that has caused the mathematician to go on and on in relentless endeavor. The infinitely small and the infinitely large have together engaged his unremitting attention through the ages. Behind all this effort, as Professor Keyser says so beautifully, there is the truly religious conviction that "transcending the flux of the sensuous universe, there exists a stable world of pure thought, a divinely ordered world of ideas, accessible to man, free from the mad dance of time, infinite and eternal. Mathematics is, in many ways, the most precious response that the human spirit has made to the call of the infinite and the eternal. It is man's best revelation of the 'Deep Base of the World.'"

Every mathematical formula represents a storehouse of information, erected by ceaseless toil. Arithmetic, geometry, and algebra have made possible this storehouse; they furnish the key that unlocks its doors and releases the treasures of the past. To comprehend, even in a rudimentary way, that this universe is a world of law; to experience the economy and joy of ordered thinking; to sense, however slightly, the eternal symmetry implanted in the geometry of form; to see how a systematic body of universal principles is discovered and demonstrated—surely this is an ethical equipment, and a training in appreciation, of incalculable importance and beauty.

Needless to say, this trilogy of aims is realized only very imperfectly in the ordinary class room. In the standard four-year

curriculum, with its one year of algebra and one year of geometry, almost exclusive attention is given to formal technique. Rarely, if ever, does the pupil suspect the existence of a rich body of applications, not to mention the higher considerations referred to above. Lack of time is the invariable excuse given for this usual poverty of content. Hence, if the junior high school is to justify itself in the eyes of the scholar and of the general public, along mathematical lines, it must so administer its subject-matter and its methods, and must make such a time allowance, that the higher mathematical aims will also have a chance to come to light.

2. *Subject-Matter.*—Mathematics originated through counting and measuring. Hence, arithmetic and geometry have become the foundations of mathematics. Both sciences have a history of thousands of years. The third branch of elementary mathematics, algebra, developed much more slowly. Its present form came into existence during the past four centuries. Yet its vast importance is inversely proportional to its youth.

The mathematics of the junior high school must, therefore, comprise the rudiments of arithmetic, geometry and algebra. These subjects must be so distributed and adjusted that the major aims of mathematical instruction are constantly kept in mind. Experience shows that the following minimum program in an academic course is feasible:

Arithmetic.—During the seventh year the work includes thorough reviews and drills in the fundamental processes, the three "cases" in percentage, with applications (profit and loss, commission, simple discount, trade discount, insurance, simple interest, bank discount) and suitable problems suggested by these topics. All this material, as outlined in the official suggestions of Director Charles E. Finch, is kept under review and is gradually extended during the eighth year. In the ninth year the effort is made to correlate arithmetic and algebra at every point, both in theory and practice.

Geometry.—The work at present comprises these topics: historical introduction, classification of forms, preliminary designs, followed by a systematic study of lines, circles, angles, triangles, polygons, parallels, symmetry, congruence, surveying, and the areas of geometric figures, both simple and composite. A spiral

arrangement of topics has been devised which secures a maximum of review and application.

The use of geometric instruments is emphasized through numerous constructions and designs, and great attention is given to clearness of perception and expression. The work at first is entirely observational and intuitive, but by slow degrees rational processes are introduced. A more ambitious program, including a study of volumes and of similarity, was outlined originally. This may still become a reality, as soon as more time is provided. It was thought wiser not to sacrifice thoroughness to breadth.

Algebra.—A brief historical introduction is followed by simple concrete exercises involving the first fundamental principles of the science. This preliminary work is to a large extent given in geometric form. It gives due attention to simple formulas and graphic methods. The subsequent study of very simple equations is conducted in a thoroughly objective way. A set of carefully graded number problems serves to connect algebra with arithmetic and geometry. Algebraic multiplication is based on a discussion of areas and volumes. The use of short processes is emphasized throughout the course. Methods of checking are taught with great care. The work of the eighth year also includes a study of square root and of the Pythagorean Theorem with suitable applications. During the ninth year the topics prescribed by the State Syllabus are completed. As far as the limited time allotment permits, the correlated type of mathematics begun during the eighth year is continued. At the end of the ninth year, therefore, the pupil is ready to begin a study of demonstrative geometry with a far more adequate preparation than can be given in the usual high-school curriculum.

Method.—If any subject depends for its vitality and the richness of its appeal on the method chosen, it is mathematics. In general, three methods may be employed in the school room. The first may be characterized as the *systematic* or *logical* method. It is comparable to the old grammatical method of studying languages. It begins with definitions and rules whose purpose is revealed to the learner only as the work progresses. Such a logical plan may be attractive to the adult mind; to the young learner it is repulsive and uninteresting because it leads him through the subject blindfolded and without a comprehen-

sion of the underlying scheme. The second method may be called the *problem* method. Its purpose is to develop the subject in connection with a carefully chosen series of progressive problems. Admirable though this procedure is, experience shows that it is too slow and rarely furnishes a sufficient basis for drill and technique. Hence a third procedure, a compromise between the first two methods, is more desirable. Its aim must be to preserve as far as possible the solid technique of the old dogmatic method, while at the same time it infuses into the subject the interest and direct motive of the problem method. In the junior high school this attempt has been made in the following manner: Each individual pupil has a notebook and the work is developed strictly on a laboratory basis. Each new principle is introduced by means of simple concrete exercises. The underlying inquiry always is, "what do *you* see?" or "what do *you* think?" In other words, the first purpose of every lesson is to secure insight and motive. When this result has been reached, the pupils enter in their notebooks a summary of the discussion, under the direction of the teacher. Then follows a series of drill exercises, continued, if necessary, on successive days until a satisfactory degree of speed has been attained. These drill exercises are either given by dictation, or are written on the blackboard by the teacher, or are contained on mimeographed sheets. At the same time every effort is made to find for each principle a suitable set of applications. Thus it is seen that the method of instruction is dominated by the three ideas of insight, skill, and appreciation.

In the junior high school supervised study has also been in operation since the opening of the school. It is too early to speak in any final way about its merits and defects. Its undoubted advantage is that it tends to develop the entire class as a unit, while at the same time it points out sharply individual differences in the rate of advance. The teacher usually knows rather definitely to what extent a lesson has been assimilated by the class. The defect of supervised study when combined with the laboratory method is the unintentional retardation of the brilliant pupil, who must wait until his slower companions "arrive," and the hopeless condition of those who are frequently absent. Both of these defects, however, are now

being remedied to a certain extent by the study coach system and by differentiated assignments.*

One other point should be noted under the heading of method. In the upper high school the various branches of mathematics are usually taught by themselves, in water-tight compartments. In the junior high school it has become possible to do far more correlating of the right kind. The early introduction of geometry especially has been a great aid in making clear algebraic principles and processes which otherwise would remain hazy. As far as possible all the important principles of mathematics are kept under review, thus insuring a greater skill in the use of all of them and an increased probability of their permanent retention. Our experience does not favor the various fusion plans which have been advocated in several parts of the country. We are still to be convinced by practical demonstration that such fusion does not ultimately lead to confusion. At any rate, a satisfactory program along the lines of the fusionists, one that will stand the test of the ordinary class-room, is still to be announced.

It is undeniable that the old high-school curriculum failed signally in not creating insight or motive, in not securing a permanent grasp of principles, methods and processes, and in ignoring—through lack of time—even the most obvious applications. All these neglected features must be cultivated by the junior high school. A beginning has been made in each of these directions. Above all, from the outset each important principle is consciously developed and all processes are treated rationally. Thus, in algebra the fundamental rules of order, grouping, and distribution are made explicit again and again. In other words, the merely mechanical juggling of symbols and figures is discouraged. By this policy we may hope to overcome that unfortunate conception of mathematics which is so ably satirized by Oliver Wendell Holmes in this biting com-

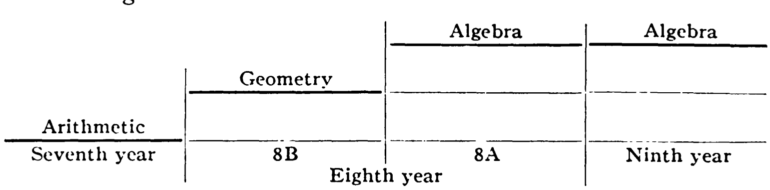
* This phase of supervised study is discussed very ably by Mr. I. M. Allen, in "Experiments in Supervised Study," *School Review*, June, 1917, pp. 398–411, although one feels strongly inclined against "auto-assignment of lessons." A good summary of the whole question of supervised study is contained in a recent article by Mr. H. C. Hines, of Iowa State University. (See "School and Society," Nov. 3, 1917, pp. 518–522).

ment: "I have an immense respect for a man of talents *plus* 'the mathematics.' But the calculating power alone should seem to be the least human of qualities, and to have the smallest amount of reason in it; since a machine can be made to do the work of three or four calculators, and better than any one of them. Sometimes I have been troubled that I have not a deeper intuitive apprehension of the relations of numbers. But the triumph of the ciphering hand-organ has consoled me. I always fancy I can hear the wheels clicking in a calculator's brain. The power of dealing with numbers is a kind of 'detached lever' arrangement, which may be put into a mighty poor watch. I suppose it is about as common as the power of moving the ears voluntarily, which is a moderately rare endowment."

Now all these details of subject-matter and method require for their fullest development a curriculum which makes adequate provision for both the proper sequence of the various mathematical topics and for the period of time which experience has shown to be necessary for their proper assimilation. This brings us to the second main aspect of our question.

II. Administrative Elements of the Question.

1. *The Curriculum.*—The way in which arithmetic, geometry, and algebra are at present distributed over the three-year period of the junior high school may be explained by the annexed diagram.*



The heavy lines indicate at what time an intensive study of each subject is begun and how long it is continued. The light lines show how long each subject is dormant or is correlated with other subjects.

* The school year is divided into two semesters. The first semester is indicated by the letter B, and the second semester by A.

In other words, geometry is now begun in the eighth year and is continued for one semester. Algebra is assigned to the second half of the eighth year and is continued for one and one half years. Historically, this sequence is the only correct one. Geometry must come before algebra. It would be a pleasure, if space permitted, to quote at length from the reports of mathematical societies and of prominent educators concerning this point. Also, it would be extremely valuable to conduct a symposium regarding the best time of introducing each subject.* Much light has been thrown on this problem by the work of the International Commission on the Teaching of Mathematics. It may be helpful to append here a few data, in tabular form, based on J. C. Brown's summary of the world's curricula in mathematics. From his study of foreign curricula, as outlined in reports of the International Commission, Mr. Brown draws comparative conclusions which it would be unfortunate to overlook at this point. He says, in part: "The European schools are doing certain kinds of work that we are not doing, some that we can not hope to do under present conditions, and some that we might not care to do if we could. They are also doing some work that we wish we could do, and some that we shall probably do before many years have elapsed.

"In every country of Europe the secondary-school period extends over at least six years. In most of the countries the majority of the teachers above the primary school have had the advantage of college or university training. The teachers have a margin of scholarship that is not common among teachers in the United States.

"Abundant provision is made for daily drill in mathematics.

* Teachers who are interested in the proper sequence of mathematical subjects should not fail to read "A Study of Mathematical Education," by the distinguished English educator Benchara Branford, published by the Clarendon Press. Dr. T. Percy Nunn, of London, whose books should be in every mathematical library, says of Branford's book that it is "the most important and original of recent English contributions to the pedagogy of mathematics." The question of the earlier introduction of geometry is discussed in Judd's "Psychology of High School Subjects," pp. 20–22, and in Superintendent Morrison's monograph on "Reconstructed Mathematics," p. 26 (see reference at end of this paper).

The educator of Europe realizes that this daily drill is absolutely necessary in order to give the pupil a real mastery of number facts and relations. A little smattering of the subject will not suffice. The pupil is expected to know thoroughly certain facts and principles, and to this end daily drill is provided. No small part of the thoroughness in detail, which is so characteristic of most of the schools of Europe, may be traced to this drill. The American pupil has some information on a great variety of topics, but much of his knowledge is vague and indefinite, rather than clear-cut notions about definite things.

"Everywhere algebra is introduced earlier than in the United States. In certain of the German schools some work in algebra is introduced during the sixth year, and in no country, except the United States, is this introductory work postponed later than the seventh school year.

"Some instruction in constructional, observational, or intuitive geometry is always offered during the sixth, seventh and eighth school years. This instruction is always of a propædeutic nature. Much emphasis is placed upon estimates and constructions.

"In all of the schools of Europe algebra and geometry are studied simultaneously during a considerable number of years. The various mathematical subjects are more closely correlated than in this country. A pupil who is studying geometry can use his arithmetic and his algebra more readily than is the case with the average American boy. The introduction of the trigonometric functions while the pupil is studying similar figures in geometry has the sanction of most of the best teachers abroad. The distinction between plane and solid geometry is much less marked than in this country. This is due, in part at least, to the fact that models are very extensively used in the study of geometry.

"Everywhere the attempt is being made to find genuine applications of mathematics that are really within the experience of the pupil and to link the subject of mathematics as closely as possible with the activities of real life. Drawing and physics are frequently taught by the same teacher, and the correlation between these subjects is found to be to the advantage of each.

"European school men believe that a course in mathematics should be planned by those who know some mathematics rather

than by educators who are practically ignorant of the subject. The reports do not indicate that the schools of Europe are hear-

TABLE I.

YEARS OF STUDY OF ARITHMETIC.

School Year.	1.	2.	3.	4.	5.	6	7.	8.	9.	10.	11.	12.	13.
Austria													
Volksschule	—	—	—	—	—								
Bürgerschule						—	—	—					
Gymnasium					—	—	—	—	—				
Realschule					—	—	—	—	—				
Realgymnasium					—	—	—	—					
England													
Elementary	—	—	—	—	—	—	—	—					
Secondary										—			
France													
Primary	—	—	—	—									
Higher primary					—	—	—				—		
Lycée					—	—	—	—					
Germany													
Volksschule	—	—	—	—	—	—	—	—					
Bürgerschule				—	—	—	—	—	—				
Gymnasium				—	—	—	—	—	—				
Realgymnasium				—	—	—	—	—					
Oberrealschule				—	—	—	—	—					

TABLE II.

YEARS OF STUDY OF ALGEBRA.

School Year.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
Austria													
Bürgerschule							—	—					
Gymnasium							—	—	—	—	—		
Realschule							—	—	—	—	—		
Realgymnasium							—	—	—	—	—		
England													
Elementary							—	—					
Secondary									—	—	—		
Private preparatory								—	—	—	—	—	
France													
Higher primary						—	—						
Lycée						—	—	—	—	—	—		
Germany													
Bürgerschule							—	—	—	—	—		
Gymnasium							—	—	—	—	—		
Realgymnasium							—	—	—	—	—		
Oberrealschule							—	—	—	—	—		

TABLE III.

Years of Study of Geometry.

School Year.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
Austria													
Volksschule					—								
Bürgerschule						—	—	—					
Gymnasium					—	—	—	—	—	—			
Realschule					—	—	—	—	—	—			
Realgymnasium					—	—	—	—	—	—			
England													
Elementary				—	—	—	—	—					
Secondary									—	—	—		
Private preparatory								—	—	—			
France													
High primary					—	—	—						
Lycée					—	—	—	—	—	—	—	—	
Germany													
Volksschule					—	—	—	—	—	—	—		
Bürgerschule					—	—	—	—	—	—	—		
Gymnasium					—	—	—	—	—	—	—		
Realgymnasium					—	—	—	—	—	—	—		
Oberrealschule					—	—	—	—	—	—	—		

ing a demand for weak algebra and anemic geometry, or even for no work in these subjects. If any pressure of this sort exists, it has hitherto produced no modification of the course of study."

2. *The Time Element.*—Compared with the above tables, our present arrangement is not ideal. Its disadvantage is that geometry is not begun early enough, that a relatively excessive amount of time is given to arithmetic, and that too little time remains for algebra. This will become clearer from a consideration of the tentative time schedule of the last two years:

Arithmetic: Seventh year : 390 minutes per week, no home lesson.
Arithmetic: Eighth year : 135 minutes per week, no home lesson.
Geometry: Eighth year (B) : 225 minutes per week, no home lesson.
Algebra: Eighth year (A) : 225 minutes per week, no home lesson.
Algebra: Ninth year (B) : 435 minutes per week, including one home lesson.
Algebra: Ninth year (A) : 480 minutes per week, including two home lessons.

* In the 9B class, one home assignment a week was allowed, and in the 9A class two home assignments were given, each estimated at 45 minutes.

A further investigation of the time allotment reveals the fact that the junior high school, under this organization, is giving hardly as much time to algebra as the upper high school in its ninth year. The following computation verifies this statement: In the upper high school (*e. g.*, East High School of this city) the available time allotment for algebra, making allowance for home study and regular interruptions, is 480 minutes per week; or, for a period of 38 weeks, a nominal allowance of *18,240* minutes. The junior high school schedule in algebra, during the past two years, is illustrated by the annexed table:

Eighth year A:	225 minutes per week.	
	19 weeks.	
	2,025 minutes.	
Ninth year B:	435 (390 + 45) minutes per week.	
	19 weeks.	
	8,265 minutes.	
Ninth year A:	480 (390 + 90) minutes per week.	
	19 weeks.	
	9,120 minutes.	
Total nominal time, not counting interruptions		21,660 minutes.
Regular interruptions (estimated)		3,960 minutes.
Actual total		17,700 minutes.

We have, then, a time schedule of 18,240 minutes in the upper high school, compared with 17,700 minutes in the junior high school.

This discrepancy is not large in itself, but it becomes serious when it is recalled that the upper high school has been entirely unable to do satisfactory work, largely because of its time element. How can the junior high school expect to do better by taking even *less* time?

It is true that supervised study combined with the study coach system is proving a help in many cases. On the other hand, the *absence* of home study in the eighth grade, with only 45 minutes a day available for preparation, presentation, and drill, is an element of retardation, since it takes slow pupils so much longer to find their way back to the territory covered during the previous lesson. Now, unless the junior high school is to cause the same mortality which is customary in the upper high school, it must gauge its progress by the attainments of the "average" pupil. Experience shows that under our present organization, *i. e.*, with supervised study, no home study, and

only 45 minutes a day in the eighth grade, the initial rate in geometry and algebra is found to be much slower than in the upper high school. Hence the apparent time budget of 17,700 minutes for algebra is actually much below the desirable minimum.

This brings up the question of remedial measures. Only two things can be done. Either geometry must be put into the seventh grade, or more time must be allowed for algebra in the ninth year. Perhaps a combination of both plans would be even better.

The consensus of mathematical educators is that geometry should be begun as early as possible, and certainly before algebra. It cannot be denied that a very heavy allowance is made for arithmetic in the seventh grade. Jessup found ("Elementary School Teacher," 1914, 461–476) that the median time spent upon arithmetic in the seventh grade of American public schools is 150 minutes; in the eighth grade, 165 minutes per week. Our own figures were stated above. Hence it should be possible, in the course of time, to provide for some geometric instruction in the seventh grade, where it belongs. The average time allotted to arithmetic in fifty representative cities is 140 hours in the seventh grade, and 142 hours in the eighth grade, or a total of 282 hours in both grades. Our total is 332 hours in the same grades of the junior high school. If this excess of 50 hours were devoted to geometry in the seventh grade, it would mean 30 minutes a day for one entire semester. Certainly this fact deserves serious attention.*

3. *Equipment.*—The junior high school cannot do its best work without a satisfactory material equipment for mathematical instruction. This includes books, models, illustrative charts, pictures, diagrams, graphs, laboratory devices and instruments,

* A new time schedule was adopted in September, 1917. Under the new plan 90 minutes a day are given to mathematics throughout the ninth year. This means the "divided period" plan. Home study is eliminated entirely. Even this new algebra schedule exceeds that of the upper high school by only two weeks; a very narrow margin, if we recall the slower initial approach and the possibility of interruptions. Hence the time element continues to be a very weighty question. In fact, the success of the junior high school is thus made to depend primarily on expert teaching and on supervised study.

a permanent display of representative notebooks and papers written by pupils, a surveying outfit, industrial and commercial products of mathematical importance, and—if possible—a projection apparatus. Much of this material is being collected now. Two interesting exhibits have occurred so far. The shops of the school furnished admirable models of transits. Many devices were improvised and each teacher is helping to contribute to the mathematical resources of the school. The time will come when all this material will have to be placed in a central distributing room, so that it can be made available to all classes. At present our greatest need is the building up of a professional library for teachers.

No regular textbook has as yet been adopted. In the eighth year no text whatever is being used by the pupils in either geometry or algebra. In the ninth year we use two supplementary texts for drill purposes and assign home lessons from the adopted high-school text. This plan is succeeding very well.

III. Social Elements of the Question.

No discussion of any feature of our work would be complete without a brief reference, at least, to the social organization of the school. Each home room has its group of officers. These officers, each group under its faculty adviser, constitute an executive body of considerable size. In this way many civic activities can be closely imitated. For one thing, a species of self-government is made possible, which is rendering the discipline of the school remarkably easy and perfect. Besides, the experiments conducted so far along the line of socialized recitations have had a quickening influence even on the stately subject of mathematics. A sincere tribute is due to the management of the school which has given unremitting attention to the details of its civic development.

1. *The Teacher.*—Next in importance to a satisfactory school organization comes, of course, the teacher. Given a good teacher, many difficulties will vanish. With a poor teacher even the best system is useless. It is a source of great gratitude that the Washington Junior High School has been blessed with a conscientious, efficient, and enthusiastic body of teachers. There has been no friction or unpleasantness of any kind. The way in

which these teachers, practically all of them from the grammar grades, prepared themselves, by means of twenty Saturday institutes, for their first pedagogic efforts in secondary mathematics, has been described officially in several reports.* Of course, all the teachers feel the urgent need of further growth in scholarship. A number of them have taken steps to supply this want privately at their own expense. It would seem that the city, in collaboration with the higher institutions of learning in our community, should offer a further training course for its teachers, at a very nominal cost. Nothing would add more to the efficiency of the schools.

The mathematical experiments of the junior high school were carried on in the following manner. The writer usually taught two classes a day, not counting days of testing and written reviews. The regularly designated class teacher, who was always present, furnished a detailed written report on each lesson taught, containing practically every question and answer, every exercise, a statement of the time occupied and of the response and attitude of the class. These reports were duplicated and distributed among the teachers. We have now complete lesson outlines of every phase of the work. They represent an invaluable set of documents, made possible by the careful, painstaking effort of the teachers.

After the first experiments had been completed, the teachers reported their own independent results in a more condensed, weekly summary. This included an estimate of the time given to the various parts of each lesson, such as review, advance, drill. Hence we have a second important set of data for further adjustments. In view of the vexing time problem referred to above these weekly abstracts were found indispensable.

A further item of general interest is the careful study that has been given to the individual response of the pupils in each of the reviews and tests. The teachers usually computed the median grade and furnished the frequency table of the class. In this way a valuable check was obtained on the difficulty of each topic and on the average deviation of each group of pupils. This made possible the application of remedial measures. Thus,

* Superintendent H. S. Weet's reports were published in *School Review,* 1916, pp. 142–151, and in *Educ. Ad. and Super.,* 1916, pp. 433–447.

two classes which were found to be distinctly bi-modal were reorganized during the past term into two more homogeneous groups, and the slower group received additional attention. Also, topics which were found too hard were given a more adequate treatment or were transferred to a later period in the course.

In short, the teachers are leaving nothing undone that is likely to make the mathematical work of the school a source of maximum benefit and interest.

2. *The Pupil.*—The Washington junior high school has a thoroughly democratic body of pupils. The foreign element predominates, the majority of the children being of Hebrew, Polish, or Italian parentage. In such a school problems of individual differences and of vocational guidance are of paramount importance. Every effort is being made to diagnose each individual case as fast as the data become available. Experience shows that the children are of average mental ability. As a rule it is not difficult to arouse their interest, but *much* harder to secure permanent retention of facts, principles, and processes. A certain dullness or mental sluggishness, noticeable in a number of the pupils, may be due to economic factors or other unfortunate home conditions. In the opinion of the writer, few things are more essential than a differentiation of pupils according to ability. Nothing whatever is gained in mathematics by hurrying over material that is not comprehended. The slow pupil often needs three and four times as much time as the brilliant one. It seems unfair to both to harness them in the same team. Careful investigations show that in the high schools of the state and of the country about 5 per cent. of the pupils may be called *excellent* (rating 90–100 per cent.), about 25 per cent. are satisfactory (rating 75 per cent.–89 per cent.), while 70 per cent. are *slow*. If the 30 per cent. whose record is good could be put into separate classes, they would do twice as much work, and the slow children, by advancing more cautiously, would get much more out of their course, too.

Fortunately, comparative statistics on the merits of differentiation are at hand. In Europe, for example, the secondary and certain elementary schools automatically exclude the children of the masses by a small tuition fee. This tends to place

a select group of pupils into the "pay schools." Without desiring to imitate that kind of differentiation, we may still be interested in the effect. The city of Breslau, Germany, recently found that the children of the Vorschulen (*i. e.*, primary schools preparing directly for the high schools) are an entire year ahead of the children in the free Volksschulen as early as the third grade. Still more startling are the figures published by the authorities of Bremen, Germany.

In the United States the most noteworthy recent experiment in differentiation along more democratic lines, and under test conditions, is that of the Speyer School, New York City. The avowed purpose of the experiment was to secure a group of promising seventh grade boys from the neighboring grammar schools of the city, who intended to elect a classical high-school course, and to enable them to complete the work of grades 7, 8 and 9 in two years. Under the auspices of Teachers College about 200 boys were thus chosen, in January, 1916. On the basis of a series of careful, preliminary tests, both psychological and educational, the boys were graded according to ability into eight groups, homogeneous as to ability. Each group was given the same work, but was encouraged to progress at its optimum pace. A visit to the institution readily impressed the writer with the scientific accuracy of the original classification. From a recent personal statement by Dr. Briggs of Teachers College I submit this passage: "The results clearly show that under such an arangement the best groups do considerably more work than the poorer ones, and that the latter, in turn, proceed without the discouragement of constant failure. For the best third of the classes one year in the three usually assigned to the junior high school is easily saved." In the sixteen years of schooling now usually provided by the combined curricula of elementary, secondary and collegiate schools, the best group could have saved three full years, thus giving them a chance to complete professional courses in the time so gained, and to begin their active life three years earlier!

Investigations in Baltimore, begun years ago under Mr. Van Sickle, point a similar lesson. Of course, a public junior high school cannot undertake too many experiments at once. It is intended for "all the children of all the people." Under no circumstance should the chief function of the junior high school

be that of a time saver. Our secondary school period is woefully short anyway. To abridge it even more would be lamentable. Richer content, clearer insight, greater appreciation, should be the goal of the junior high school, rather than a reduction of time. The danger of destroying the chief purpose of the junior high school by making it primarily an economy expedient, was clearly anticipated by Dr. J. Sachs, of Teachers College, Columbia University. In his admirable book on "The American Secondary School" (Macmillan Co., 1912), he quotes President Pritchett as saying: "The real struggle in the American higher school is between that influence which makes toward thoroughness and that which makes toward superficiality; and if the high school is to become the true training place of the people, the ideal of thoroughness must supplant the ideal of superficiality" (5th Annual Report Carnegie Foundation, 1910, p. 64). The author then continues as follows: "Fatal of course to the introduction of the six-year high-school scheme would be the assumption that with this increase in the number of high-school years the high school could blithely undertake the functions of the first two college years; we cannot protest too energetically against such an endeavor, for it would again stimulate the substitution of superficiality for thoroughness. If once it could be established that the work of the high-school stage were being done too well, then there might be a pretext for this fatuous clamor" (see p. 112, where additional references are also given).

But as soon as opportunity and experience make it feasible, it may be worth while to think of the accelerated pupil. True democracy involves the free and spontaneous development of each God-given native talent. Without differentiation a shortening of the secondary period of education is impossible, and many a fine mental endowment cannot be developed. The literature bearing on this subject, especially on practical classroom adjustments made necessary by differences in the capacity of pupils, is rapidly increasing, A brief recent summary, together with a bibliography, may be found in S. Ch. Parker's "Methods of Teaching in High Schools," Ginn & Co., 1915, pp. 362–390. Of course, Thorndike's fundamental studies of individual differences will continue to offer helpful suggestions along this line

(Thorndike, E. L., "Educational Psychology," Teachers College, 1914, Vol. III., pp. 142–388).

3. *The Community.*—In a democracy like ours no public-school issue can be considered apart from the life of the community. Our schools are supported by all the people and they must serve all the people. There is no type of school that is better adapted to render this service than the junior high school. Its differentiated courses, its departmental work, its school day, its laboratories and shops, its entire organization together offer a far more genuine educational opportunity than the ordinary grammar school.

Without doubt this fact is being recognized more and more. Not only are all the public exercises of the school well attended by our citizens, but an increasing number of out-of-town visitors furnishes evidence of the general interest in the development of the 6–3–3 plan.

Now, the work of the school could be strengthened very considerably by keeping in mind the real problems and applications suggested by the affairs of our big industrial community. This is especially true of mathematics, for again and again we must think in "quantities, proportions, forms, and relationships." Steps have been taken, with the co-operation of the chamber of commerce, to secure community data for the mathematics of the schools. A valuable beginning has been made. A number of the city departments have furnished splendid graphic material, and some of the great business concerns have contributed problems of interest. It is hoped that this work may be continued. Nothing attracts the children so much as concrete situations of a local kind. If mathematics is ever to "function" in the life of the adult, it must be accomplished through such early contact between theory and practice.

Summary and Conclusion.

How shall we reply, in conclusion, to the natural queries of the impartial critic and observer: Is the junior high school making good? And what may be done to render its future even more promising?

It is too early to give a final answer to these questions. Too many unsettled and undefined elements enter into the issue. We can judge only from the measurable features of the work of

two years. These certainly justify an optimistic spirit. The reports show that the holding power of the school is extremely gratifying, that in all departments the percentage of elimination, of failure or non-promotion means a distinct gain over former conditions.

In the mathematical classes there is definite evidence of both interest and ability to do the work. A number of pupils recently mentioned algebra as their easiest subject. If the time element can be modified as suggested above, there is little doubt that very few pupils will fail to derive both increased benefit and pleasure from this "dry" subject. Hence there is every prospect that the curriculum of the junior high school will help to solve many difficulties that have been so baffling in the upper high school.

Much remains to be done, especially in the ninth year. However, our experiments are sufficiently conclusive to warrant the compilation of a detailed course of study. This is now in preparation and in due time will be available for teachers and pupils.

The junior high school will certainly bring the world's mental and spiritual treasures nearer to the people. It will invite the young students to share the riches of the past and to become productive workers themselves. Thus it will carry its cultural mission into a multitude of homes.

As to mathematics, it will remove many an old prejudice. Perhaps it can even produce some of the enthusiasm which we find in the life of the great thinkers. In that case there will be even greater cause for the glowing outlook pictured so charmingly by one of the distinguished writers named previously:

"The golden age of mathematics—that was not the age of Euclid, it is ours. Ours is the age in which no less than six international congresses of mathematics have been held in the course of nine years. It is in our day that more than a dozen mathematical societies contain a growing membership of over two thousand men representing the centers of scientific light throughout the great culture nations of the world. It is in our time that over five hundred scientific journals are each devoted in part, while more than two score others are devoted exclusively, to the publication of mathematics. . . . It is in our time that as many as two thousand books and memoirs drop from

the mathematical press of the world in a single year, the estimated number mounting up to fifty thousand in the last generation.

"It is not, however, by such comparisons nor by statistical methods nor by any external sign whatever, but only by continued dwelling within the subtle radiance of the discipline itself, that one at length may catch the spirit and learn to estimate the abounding life of modern mathesis: oldest of the sciences, yet flourishing today as never before, not merely as a giant tree throwing out and aloft myriad branching arms in the upper regions of clearer light and plunging deeper and deeper root in the darker soil beneath, but rather as an immense mighty forest of such oaks, which, however, literally grow into each other so that by the junction and intercrescence of limb with limb and root with root and trunk with trunk the manifold wood becomes a single living organic growing whole."

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